Mathematical modelling of the relationship between terrestrial LIDAR scan point density and volumetric assessment of underground cavities

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Abstract

Terrestrial LIDAR (T-Lidar, or 3-D) scanning gives outstanding detail in cave surveying, generating extremely large datasets of dense point clouds, resulting in very detailed and precise 3D models of the scanned caves. These models are commonly used to determine the volume of chambers.

Intuition tells us that the denser the point cloud, the better it will fit the real dimensions of the cave. Here we prove that this is not the case. We show that with a low number of measured points it is possible to calculate volumes which will match the true volumes of a cavity with high precision.

Scanning at extremely coarse resolution with angles as high as 4.3° (approximately 1/400 of full resolution) gives a good estimate of volume, although detail is not rendered. The linear relationship between the distance of the scanner and the scanned resolution of the cave wall indicates that < 1 cm-scale detail can be rendered by scanning at ¼ resolution up to ~20 m distance. For the same detail, at distances between ~20 and ~70 m, scanning at higher resolution will be required. It is not possible, even with full resolution, to get centimetre-scale detail at distances greater than ~70 m. Therefore, it is apparent that scanning caves at only ¼ resolution is generally quite sufficient to represent the real volume of the cave and most of the detail.

Keywords:

1. Introduction

Terrestrial LIDAR (T-Lidar, or 3-D) scanning is increasingly being used for surveying caves, with remarkable results (Buchroithner 2015; Gallay et al. 2015; McFarlane 2013; McFarlane et al. 2013). The laser measurements generate immensely dense point clouds that can be used to generate very detailed and precise 3D models of the scanned caves. A common goal of 3D modelling of caves is the determination of cavity volume (including those recognised as, or claimed as, the largest natural underground cavities on Earth). Intuition tells us that the denser the point cloud, the better it will fit the real dimensions of the cave, and thus the better the estimate of volume will be. This concept is used in many discussions about the precision of volume determination by terrestrial LIDAR. Here, we test this assumption by considering how well the generated 3D model is likely to match the real scanned volume of the cave, represented here by a theoretical structure whose volume can be calculated precisely.

2. Determining the fit of a triangulated sphere in the circumscribing sphere

Imagine a perfect sphere with a given radius that precisely contains a Platonic solid with the same centre as the sphere. Each vertex of this regular polyhedron must be on the surface of the sphere. To keep it simple, we only consider polyhedrons that solely contain identical equilateral triangular faces (Figure 1). This is acceptable because triangles are one of the basic building elements of a polygonal mesh. From the five Platonic solids only three fit this requirement: the tetrahedron, the octahedron and the icosahedron.

The volume with the lowest face count possible is a tetrahedron made from only 4 triangles and 4 vertices, but it does not resemble a sphere. If we add more triangles we get an octahedron with 8 faces and 6 vertices. The next step is an icosahedron with 20 faces and 12 vertices. The icosahedron resembles a sphere already, but still more triangles are needed. However, it is mathematically not possible to add more equilateral triangles. We cannot create a new volume by adding a sixth triangle around one of the vertices of an icosahedron. This operation will only result in a flat face. The trick is to divide the faces of the icosahedron into 4 new triangles and make sure that the new vertices are contained on the circumscribed sphere. This can only work with isosceles triangles instead of equilateral ones.

This can easily be demonstrated with 3D computer graphics software, such as, for example, the open-source software Blender (https://www.blender.org/). With Blender it is possible to create so called “icospheres” – the name for a polyhedral sphere made up of triangles. During the creation of an icosphere the user can specify a number of subdivisions. Each increase of subdivision splits each triangle face into four triangles. A subdivision with n = 1 generates the already-mentioned icosahedron. A subdivision with n = 5 creates an icosphere with 6820 faces, which results in a nearly perfect sphere (Figure 2).
We compare the surface area \( S_{\text{sphere}} \) and volume \( V_{\text{sphere}} \) of a real sphere with the surface area \( S_{\text{ico}} \) and volume \( V_{\text{ico}} \) of the generated icospheres. Blender has specific functions to calculate these values.

In the calculations, the following symbols and equations are used:

- \( S_{\text{ico}} \): Surface area of icosphere (m²) calculated by Blender
- \( V_{\text{ico}} \): Volume of icosphere (m³) calculated by Blender
- \( d \): Average length of the sides of the triangles of the icosphere (m) calculated by Blender
- \( \Delta S \): Difference between surface areas \( S_{\text{sphere}} \) and \( S_{\text{ico}} \) (m²)
- \( \Delta V \): Difference between volumes \( V_{\text{sphere}} \) and \( V_{\text{ico}} \) (m³)
- \( \Delta S\% \): Percent difference between surface areas \( S_{\text{sphere}} \) and \( S_{\text{ico}} \)

\[
\Delta S\% = \frac{\Delta S}{S_{\text{sphere}}} \times 100
\]

\( \Delta V\% \): Percent difference between volumes \( V_{\text{sphere}} \) and \( V_{\text{ico}} \)

\[
\Delta V\% = \frac{\Delta V}{V_{\text{sphere}}} \times 100
\]

\( \text{Res} \): Calculated resolution angle in degrees (°) based on \( R \) and \( d \) (Figure 3)

\[
\text{Res} = 2 \sin^{-1} \left( \frac{d}{2R} \right)
\]

Assume we are in a perfectly spherical cave chamber with a radius, \( R \). In the centre of this void the T-Lidar scanner is positioned. From the distance, \( d \), between two adjacent scan points of the resulting point cloud and the radius, \( R \), of the sphere, the resolution angle of the scanner can be calculated using equation 3.

The test case (Table 1) is a sphere with radius, \( R \), of 1.00 m. This sphere has a surface area, \( S_{\text{sphere}} \), of 12.566 m² and a volume, \( V_{\text{sphere}} \), of 4.189 m³. The results can be applied to a sphere of any radius. Suppose we have a spherical cave room with a radius, \( R \), of 100 m and this room is scanned with a device using a resolution angle of 4.30° (last line of Table 1). The distance between two neighbouring scan points on the cave wall will be \( d = 7.5 \) m.

It is remarkable to see that this coarse resolution results in an extremely low difference between the real volume and the scanned volume: \( \Delta V\% = 0.23 \% \). Of course, commercial T-Lidar scanners have a much smaller resolution angle, at least two orders of magnitude better than 4.30°. For example, the Faro Focus 3D X 330 has a resolution of 0.009° or 15 mm at 100 meters distance, or 478 times better than the 4.30° in our example. We can conclude that volume determinations based on the results from a commercial scanner are more than adequate to represent the real volume of the cavity.

Although scanning with a resolution angle as big as 4.30° might result in good volume estimations, the resulting scans will lose all the detail of real world caves. In the next section we look more closely into the relation between scan resolution and the detail obtained.

### 3. Relation between scan resolution and detail

The Faro Focus 3D X 330 has a resolution of 40960 points over 360°, which can be translated to a resolution angle of 0.009°. A full scan like this takes rather a long time and produces a point cloud of 711 million points. So, in most cases, while scanning in the field, a \( \frac{1}{4} \) resolution is used, with 10240 points over 360° or 0.036°, which results in clouds of 44 million points.

Using these two resolutions (0.009° and 0.036°) we can calculate the distance (in m) between two adjacent scan points, \( d \), as a function of the distance between the scanner and the cave wall, \( R \). We can rewrite equation 3 to get the value \( d \).

\[
d = 2R \sin^{-1} \left( \frac{\text{Res}}{2} \right)
\]
There is a linear relation between the distance of the scanner and the scanned resolution of the cave wall. This relationship must be taken into consideration before starting an underground scanning campaign. For example, if centimetre-scale resolution is required (as, for example, might be the case when scanning bat roosts; cf. Azmy et al., 2012) and the scanner is set at ¼ resolution, then the instrument should not be placed more than 20 metres from the walls. If centimetre-scale resolution is required for high cave ceilings, it might be important to work at full resolution. However, as figure 4 shows, it is not possible, even with full resolution, to get centimetre details for ceilings that are higher than 70 m.

4. Conclusion

Here, we have shown the falseness of the intuitive concept that higher resolution should yield more accurate estimates of cave volume. Scanning caves with commercial T-Lidar scanners, even at only ¼ resolution (i.e., a resolution angle of 0.036°), is generally quite sufficient to represent the real volume of the cave and most of the detail. At ¼ resolution, detail at <1 cm scale can be achieved up to a distance of ~20 m from the cave wall. Scanning at full resolution (i.e., a resolution angle of 0.009°) would render detail at <1 cm scale up to a distance of ~70 m. If volume were the only consideration, then scanning at a resolution an order of magnitude lower would suffice, but much of the detail would be lost.

Therefore, it is clear that before starting a scan project in the field it is important to consider the required resolution of the final cave model. The maximum distance between the scanner and the cave walls depends on this resolution. High resolution cannot always be obtained in caves with high ceilings or over long distances. Scanning at ¼ resolution at distances over 150 m becomes increasingly inaccurate. Therefore, for most caves, it is advisable to delete all scan points which are farther away than ~100 m from the scanner before registering the scans.

References


