

# Stick balancing: On–off intermittency and survival times

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**Abstract** The fluctuations in the vertical displacement angle of a stick balanced at the fingertip exhibit on-off intermittency. However, even a skilled balancer cannot indefinitely maintain a stick balanced at their fingertip. The survival function for stick balancing,  $P(t_{\text{esc}} > t)$ , is shown to have the form of a Weibul function,  $\exp(-\lambda t)^\beta$ , where  $\lambda$  is a constant and  $\beta > 1$ . The measured survival function can be reproduced by a stochastic delayed discrete map possessing only unstable solutions. These observations emphasize the importance of state-dependent, or parametric, noise in this balancing task.

## 1 Introduction

Two ubiquitous properties of the nervous system are the presence of time delays and random, uncontrolled fluctuations (“noise”). Presumably spontaneous fluctuations reflect the dynamics of underlying stochastic dynamical systems with retarded variables. It follows that the analysis of the fluctuations generated by such dynamical systems should provide important non-invasive insights into how the nervous system works, how the abnormal dynamics associated with disease arise (e.g. seizures, abnormal movements, falls), and how therapeutic strategies might be designed.

Two recent developments have attracted renewed attention into studies of statistical neurodynamics. First, it is increasingly becoming recognized that neural noise is state-dependent, or parametric, i.e. the effects of noise depend on the state of the system [4, 7, 16, 18, 28, 36, 43]. Consequently, mathematical models

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for neurodynamics take the form of stochastic delay differential equations of the form, for example,

$$\dot{x}(t) = f(x(t - \tau), \xi(t)) \quad (1)$$

where  $f$  is a nonlinear function,  $\tau$  is the time delay,  $\xi$  describes the noisy inputs, and  $\dot{x}$  is the first derivative of  $x$ . Very little is known about the properties of (1) (for notable exceptions see [14, 23, 25, 32]). Second, measurement devices with high spatial (microns) and temporal (milliseconds) resolution have become routinely available at both the benchtop and the bedside. With better data has come the realization that power laws arise quite frequently in physiological dynamics (e.g. [35]) and fluctuations once thought to be Gaussian distributed are, in fact, better described by other types of  $\alpha$ -stable statistical distributions, such as the Lévy distribution [26].

Dynamical systems containing power laws and Lévy distributed variables are difficult to study experimentally because very long data sets (e.g.  $10^5 - 10^6$  data points) are required to obtain “good statistics” [42]. In biological preparations sufficiently large data sets often require many hours to days to collect. Problems arise because biological preparations themselves change over these time scales. For progress to be made alternate avenues must be identified so that important insights can be drawn from smaller data sets. Power laws arise in the fluctuations of a stick balanced at the fingertip [7, 13, 39, 40]. Here we show that stick balancing can also be studied within the context of a survival, or first passage, time problem. This approach may offer a more accessible route for unraveling the nature of the neural control of this balancing task.

## 2 Background

### 2.1 Stick balancing

The study of stick balancing at the fingertip is important for understanding the neural control of balance [7, 13, 27, 38, 39, 40] and skill acquisition [8, 9]. There are a number of advantages offered by the study of stick balancing at the fingertip as a paradigm for the neural control of a skilled movement: 1) the movements can be measured with high precision using 3-D motion analysis techniques [7, 8, 9]; 2) mathematical models for the control of inverted pendulum with time-delayed feedback have been extensively studied [11, 15, 22, 37, 38]; 3) skill level can be objectively determined by, for example, measuring the fraction of trials that can be balanced longer than a certain time [8, 9]; and 4) skill level can be significantly increased with just a few days of intensive practice [8, 9].

The importance of the time delay for stick balancing is demonstrated by the observation that longer sticks are easier to balance than shorter ones: once the stick becomes sufficiently long its rate of movement becomes slow relative to the time required by the nervous system to make corrective movements. Indeed mathematical models for the stabilization of an inverted pendulum with time-delayed feedback suggest that in order to balance a stick of length,  $\ell$ , the time delay must be less than a critical value proportional to  $\sqrt{\ell}$  [37, 38].

Our focus is on the effects of parametric noise on the control of stick balancing at the fingertip. Therefore we studied the situation in which the fingertip is allowed

to move freely in three dimensions while the subject was seated with their back against the back of a chair [7, 8, 9]. During stick balancing, skilled individuals hold their fingers and wrist rigid and movements occur mainly at the elbow and shoulder. It is well known that when the pivot point of an inverted pendulum moves, either periodically [1] or noisily [2, 3], the perturbations enter the equations of motion through their effects on a parameter.

## 2.2 On-off intermittency

The controlled variable for stick balancing can be considered to be the vertical displacement angle,  $\theta$  [7, 13, 39, 40]. It is important to note that  $\theta$  is a function of only the difference,  $\Delta z$ , in the vertical coordinates of the upper and lower ends of the stick:  $\cos \theta$  is equal to  $\Delta z/\ell$ . Figures 1a,c shows the temporal series for the changes in  $\Delta z/\ell$ . It has long been recognized that the fluctuations in  $\Delta z/\ell$  alternate between periods dominated by small fluctuations and those in which larger changes occur [7, 39, 40]. Cabrera and Milton [7] were the first to recognize that these characteristics of the fluctuations in  $\theta$  were the result of a stochastic phenomena known as on-off intermittency. We first discuss how the presence of on-off intermittency can be demonstrated in the fluctuations in  $\theta$  and then briefly review its importance for the control of movement.

The first step to demonstrate the presence of on-off intermittency is to choose a threshold (horizontal white line) (Figure 2). As pointed out in [7] the position of this threshold is arbitrary; however, it should be placed in such a way that there are a sufficient number of threshold crossings to permit good statistics. Next the time intervals,  $\delta t$ , between successive crossings of this threshold in the upward (i.e. corrective) direction are measured. These time intervals are the laminar phases. Figures 1b,d show a double log plot of the normalized probability of having laminar phases of length  $\delta t$ ,  $P(\delta t)$ , versus  $\delta t$ . The fact that this plot is linear strongly implies that the fluctuations in  $\theta$  are governed by a power law with exponent  $-\frac{3}{2}$ . An exponent of  $-\frac{3}{2}$  is a characteristic of on-off intermittency. Furthermore the power spectrum for the fluctuations in  $\theta$  exhibit a  $-1/2$  power law [7]. This is also a characteristic of systems that exhibit on-off intermittency [44]. Both of these power laws are intrinsic components of the neural control mechanism for stick balancing since they disappear when the subject closes their eyes and moves their fingertip/arm in a fashion that mimics the movements during stick balancing. On-off intermittency is a phenomenon that arises because of the stochastic or chaotic forcing of an important control parameter across a stability boundary [21, 33]. There are a number of important insights that the demonstration of on-off intermittency provides into the nature of the neural control of stick balancing. First, on-off intermittency strongly argues that the parameters for the feedback mechanism for the neural control of stick balancing must be tuned to be close to a critical threshold; the proximity to the stability boundary depending on the intensity of the noise [17]. Thus the presence of on-off intermittency is consistent with the hypothesis of Kelso and colleagues (for a review see [20]) that the neural control mechanisms for a variety of manual movements are tuned at the edge of stability. In addition, since many neurophysiological systems are stabilized at the edge of stability, e.g. [20, 28, 30], we anticipate that on-off intermittency

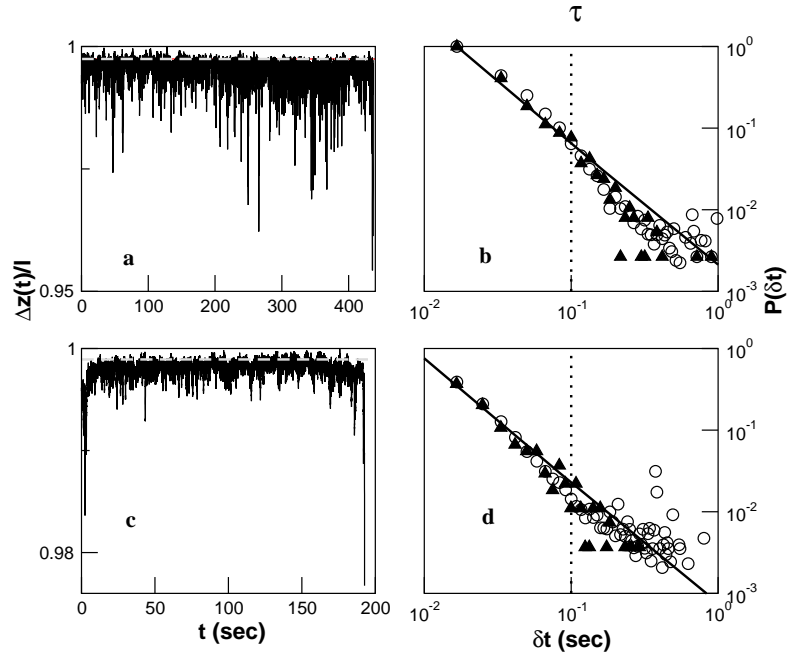


Figure 1: Temporal series for  $\frac{\Delta z}{l}$  for a) 39 cm and c) 62 cm stick balanced at the fingertip. The horizontal white-dashed line depicts the threshold position. The corresponding log-log plots for the laminar phase probability distribution,  $P(\delta t)$ , are shown in, respectively, b) and d). The vertical dotted line shows the latency for stick balancing estimated from the cross-correlation for the movements of the tip of the stick versus those of the fingertip. The triangles ( $\blacktriangle$ ) are calculated from a single realization and the circles ( $\circ$ ) represent the mean distribution obtained from, respectively, b) 89 and d) 48 realizations. The solid line represents a power law with exponent  $-3/2$ . Similar results were obtained for five other subjects (three males and two females) using stick lengths from 39-149 cm. These experiments were performed according to the principles of the Declaration of Helsinki and informed consent was obtained. Experimental protocols were approved by the institutional review board of The University of Chicago Hospitals.

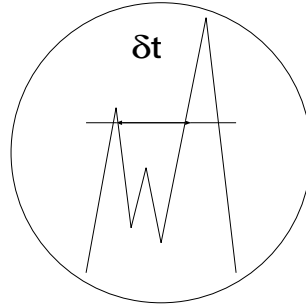


Figure 2: Schematic representation illustrating the measurement of a laminar phase,  $\delta t$ .

should be a commonly observed property of neural control. Second, on-off intermittency provides a mechanism that enables corrective movements to be made on time intervals shorter than the neural delay ( $> 98\%$  of the  $\delta t$  for human stick balancing are shorter than the delay; represented by the vertical dotted lines in Figure 1b,d). Third, on-off intermittency provides a mechanism of non-predictive control for stick balancing, i.e. the fluctuations in  $\theta$  resemble a random walk for which the mean value of  $\theta$  is approximately zero, i.e. the upright position has been statistically stabilized [7]; and finally 4) the existence of a non-predictive control mechanism for stick balancing reduces the demands on consciously directed control movements and thus “frees up” the conscious and attention mechanisms of the brain for other tasks [9, 10].

### 2.3 Modeling efforts

The equation of motion for an inverted pendulum stabilized with delayed feedback is

$$m\ddot{\theta}(t) + \gamma\dot{\theta}(t) - k \sin \theta(t) + F(\theta(t - \tau)) = 0 \tag{2}$$

where  $m$  is the mass of the stick, and  $\tau$  is the time delay, and  $F$  describes the time-delayed feedback, i.e.

$$F(\theta(t - \tau)) \approx r_0\theta(t - \tau) + r_1\dot{\theta}(t - \tau) + \dots$$

and  $\gamma, k, r_0, r_1$  are constants. Equation (2) indicates that the movements of the stick are governed by three forces, namely, its weight, friction, and a restoring force applied at the fingertip that depends on the value of  $\theta$  at a time  $t - \tau$  in the past. The “-” sign arises because we have taken  $\theta = 0$  to be the upright position. For appropriate choices of the parameters, (2) possesses a stable upright position [7, 37, 38].

Cabrera and Milton [7] demonstrated that the statistical properties of stick balancing can be reproduced by introducing parametric noise into (2) by assuming that the gain of the feedback function varies noisily, i.e.

$$F(\theta(t - \tau)) \sim r_0(t)\theta(t - \tau) \tag{3}$$

where  $r_0(t)$  is approximated by a random function:

$$r_0 \rightarrow r_0 + \xi(t) \equiv r_0(t) \quad (4)$$

and  $\xi(t)$  is zero mean white noise, i.e, a Markovian stochastic process of intensity  $D$  and correlation function,

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t-t') \quad (5)$$

With these considerations, (2) becomes

$$m\ddot{\theta}(t) + \gamma\dot{\theta}(t) - k\sin\theta(t) + (r_0 + \xi(t))\theta(t - \tau) = 0 \quad (6)$$

In principle the feedback would be expected to be a function of both  $\theta(t - \tau)$  and  $\dot{\theta}(t - \tau)$  [11, 15, 27, 37, 38]. However, it has been possible to reproduce the statistical properties of the fluctuations in  $\theta$  by using feedback that depends only on  $\theta(t - \tau)$  [7].

## 3 Results

### 3.1 Survival times

There is an important distinction between stick balancing at the human fingertip and the stabilization of an inverted pendulum using a mechanical device with time-delayed feedback (e.g. [22, 37, 38]). A mechanically stabilized inverted pendulum remains upright as long as the control is applied. From a dynamical systems point of view this focusses attention on the identification of the attractor and its properties [11, 15, 22, 27, 37, 38]. In contrast, a stick, even a long one (e.g. 149 cm), balanced at the fingertip can be maintained only for a few minutes even by a skilled stick balancer. The fact that the stick falls has not been explicitly considered in previous studies of stick balancing [7, 27, 39, 40], but is obviously an essential feature of this task. Considerations of why the stick falls shifts the attention of modeling efforts towards considerations of survival, or first passage, time problems.

The survival function,  $P(t_{\text{esc}} > t)$ , for stick balancing can be estimated by performing a large number of trials (in practice 50 – 100) and calculating the fraction of trials for which the stick is still balanced as a function of  $t$  [24]. Figure 3 shows the survival function measured in this way for a single subject using four different stick lengths with the same mass. Two observations suggest that the survival statistics for stick balancing are related to the intrinsic dynamics of the balancing task and do not simply reflect, for example, that the subject is becoming tired or less attentive to the task: 1) the shape of  $P(t_{\text{esc}} > t)$  is qualitatively the same for all stick lengths from 39-149 cm; and 2) the survival statistics for 39-62 cm sticks are measured over the time interval during which most 149 cm length sticks are balanced.

The survival functions in Figure 3 have the form of a Weibul survival function, i.e.

$$P(t_{\text{esc}} > t) \approx \exp(-\lambda t)^\beta \quad (7)$$

with  $\beta > 1$ , where  $\beta, \lambda$  are constants. Weibull survival functions with  $\beta > 1$  commonly arise in the analysis of failure times (e.g. [19]) and in patient survival statistics for certain kinds of cancer [24].

### 3.2 Discrete-time model for stick balancing

From the point of view of (6) the stick falls because the intensity of the parametric noise is of the same order as the dimensions of the basin of attraction in parameter space. Thus there is a finite probability that the trajectory escapes the basin of attraction for the upright position and the stick falls. An alternate interpretation is that the upright position is unstable and is stabilized transiently by the action of a random and delayed restoring force. In this case the stick falls because the random force becomes temporarily insufficient. A simple way to derive a model of

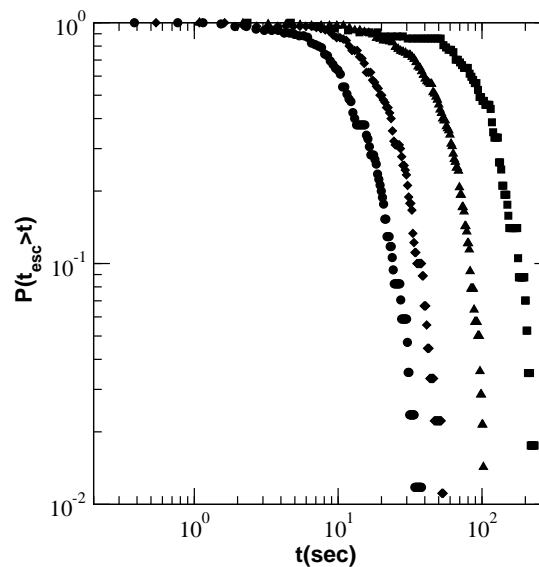


Figure 3: Survival function for stick balancing for four different stick lengths (39 cm ( $\bullet$ ), 55 cm ( $\blacklozenge$ ), 62 cm ( $\blacktriangle$ ), 141 cm ( $\blacksquare$ )). The survival function,  $P(t_{\text{esc}} > t)$ , was determined by measuring the fraction of trials still balanced at time  $t$  as a function of  $t$  (number of trials  $> 50$  in all cases). Data is shown for a single subject. The mass of the sticks was 35 g (different materials having differing densities were used in order to keep the mass constant). A total of seven subjects were studied in this manner using stick lengths from 38-149 cm. In all cases the survival function had the same qualitative shape as shown here. However, there was considerable variation in the balancing skill level between subjects: for example, the percentage of trials lasting longer than 20 s for a 62 cm stick ranged between subjects from 0 to 100 %.

this type is to consider the movements of a mass under the influence of a spring in which there is a random force that acts to maintain the mass near one of the

unstable positions of maximal elongation. It is straightforward to show that the equation of motion is

$$m\ddot{\theta}(t) + \gamma\dot{\theta}(t) + k \sin \theta(t) + (r_0 + \xi(t))\theta(t - \tau) = 0 \quad (8)$$

The survival time for (8) is the probability that a stick balanced at time  $t$  is balanced at time  $t + \Delta$ , where the time interval  $\Delta t$ , corresponds, for example, to the temporal resolution of the measuring device. Thus it is appropriate to analyze the survival times generated by (8) within the context of the corresponding discrete time map [5, 6, 24].

In order to obtain the corresponding discrete time map we rewrite (8) as

$$\begin{aligned} \frac{dx}{dt} &= \frac{x(t+dt) - x(t)}{dt} = y(t) \\ \frac{dy}{dt} &= \frac{y(t+dt) - y(t)}{dt} = -\frac{\gamma}{m}y(t) - \frac{k}{m} + \frac{r(t)}{m}x(t - \tau) \end{aligned}$$

or

$$\begin{aligned} x(t+dt) &= x(t) + y(t)dt \\ y(t+dt) &= y(t) - \left\{ \frac{\gamma}{m}y(t) + \frac{k}{m}x(t) - \frac{r(t)}{m}x(t - \tau) \right\} dt \end{aligned} \quad (9)$$

Now apply the following temporal transformations:

$$\begin{aligned} \frac{t}{dt} &\rightarrow t' \\ \frac{t+dt}{dt} &\rightarrow t' + 1 \\ \frac{t - \tau}{dt} &\rightarrow t' - h \end{aligned}$$

Provided there exist a integer number  $h$ , (9) can be written as the delayed map

$$\begin{aligned} x_{n+1} &= x_n + y_n \\ y_{n+1} &= \left(1 - \frac{\gamma}{m}\right)y_n - \frac{k}{m}x_n + \frac{r_n}{m}x_{n-h} \end{aligned} \quad (10)$$

where  $x_{n+1}, x_n, x_{n-h}$  are respectively the values of  $x$  at times  $n, n+1$  and  $n-h$ , where  $h$  is an integer denoting the delay,  $y_n, y_{n+1}$  are the values of  $y$  at times  $n$  and  $n+1$ , and

$$r_n = r_0 + \xi_n$$

where  $r_0$  is a constant and  $\xi_n$  is a random variable chosen from a Gaussian distribution of discrete variables having mean equal to zero and variance,  $\sigma$ .

The survival function,  $P(t_{esc} > t)$ , predicted by (10) can be computed as follows: The parameters in (10) are adjusted so that the solution escapes a defined

interval at some time if the initial position was equal to the maximal elongation. This corresponds to stick falling. Thus, in exactly the same way that  $P(t_{esc} > t)$  is measured for stick balancing (Figure 3), we can determine  $P(t_{esc} > t)$  from (10) by computing multiple realizations.

Figure 4 compares the survival time statistics of (10) for  $h = 1$  to those observed for a subject balancing a 62 cm stick with mass 35 g. As can be seen the agreement is quite good. The properties of this map when the delay is not zero and in the presence of parametric noise are currently under investigation.

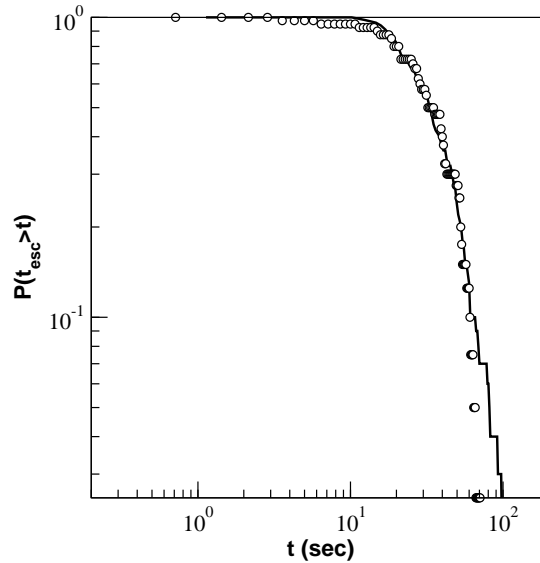


Figure 4: Comparison of the  $P(t_{esc} > t)$  generated by (10) (solid line) to that measured experimentally ( $\circ$ ). Parameters:  $\ell = 62$  cm,  $m = 35$  g,  $k = 35$ ,  $\gamma = 40 \text{ sec}^{-1}$ ,  $r_0 = 0.97$ ,  $\sigma = 0.15$ .

## 4 Discussion

Here we have shown that the survival times for stick balancing can be reproduced by a simple discrete time model that contains time-delayed feedback with a noisy gain. Measurement of the vertical displacement angle requires the use of motion analysis techniques that have sufficient temporal and spatial resolution. Presently such equipment is quite expensive and hence available to only a few laboratories. However, survival times for stick balancing can easily be estimated using just a stop watch. Thus measurements of stick balancing survival times may offer a more accessible route for unraveling the nature of the neural control of this balancing task.

It is important to realize that the same data used to measure the survival times for stick balancing, when viewed in another way, yields the power laws characteristic of on-off intermittency (e.g. Figures 1b,d). The fact that the discrete

map that reproduces the observed survival times possesses only unstable solutions suggest that this may also be true for the corresponding stochastic delay differential equation, i.e. (8), for appropriate choices of the parameters. However, in order to be logically consistent, this line of reasoning implies that (8) may also produce the power laws of on-off intermittency even though there are no stability boundaries. Little attention has been given to the possibility that on-off intermittency in stochastic dynamical systems may reflect fluctuations around an unstable state. The significance of this possibility is that it would readily explain how power laws characteristic of on-off intermittency occur even though other phenomena expected to arise at a stability boundary, such as slowing down phenomena, may not. We are currently working on the analysis of (8).

A simple mechanism to generate a Weibull survival function (e.g. (7)) with  $\beta > 1$  arises in the case of 1-D maps that generate a stationary density [24]. In these models a trajectory is assumed to escape from the unit interval whenever it enters an arbitrary sub-interval. The case  $\beta > 1$  corresponds to the situation in which the size of the sub-interval enlarges as a function of time. Our observations suggest that this type of survival function can also arise in 2-D maps as a consequence of the effects of state-dependent, or parametric, noise.

The tendency of present-day bio-mathematicians to investigate deterministic models of physiological processes under-emphasizes the fundamental importance of careful attention to the statistical properties of the observed fluctuations in physiological variables. At the very least the statistical properties of the data must be reproduced by a proposed model; if not, then it is difficult to accept the model as a valid description of the physiology. However, the importance of statistical neurodynamics is not simply limited to model justification. Historically, statistical neurodynamics was the first to draw attention to the presence of ion channels, or pores, in neuronal membranes [45], to the nature of spike generating mechanisms [41], to the importance of noise for the detection of weak sensory signals [31], to distinguish deterministic (i.e. chaotic) from stochastic mechanisms [34], and, as illustrated in this communication, to provide a powerful tool to delineate the nature of the underlying control mechanisms. Moreover, the scientific value of statistical neurodynamics is likely to increase in importance, rather than diminish. The recent trends towards ever more burdensome regulations and restrictions for research on human subjects implies that studies that focus on non-invasive measurements will soon dominate, particularly in the context of normal physiology. A key point is that the interplay between noise and nonlinear control mechanisms can produce unexpected and surprising dynamics, especially if time delays are present [7, 12, 29, 32].

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