Υ dissociation in a quark-gluon plasma

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I consider the dissociation of Υ due to absorption of a thermal gluon. I discuss the dissociation rate in terms of the energy density, the number density, and the temperature of the quark-gluon plasma. I compare this to the effect due to screening.

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When the bottom quark or antiquark is struck by a high-energy gluon, the upsilon meson can dissociate into other elements. The medium, the quark-gluon plasma, can be full of gluons that can cause this dissociation, and this can happen by exciting color-singlet $|b\overline{b}\rangle^{(1)}$ into a color-octet continuum state. The bottom quark absorbs energy from the gluon field. When the bottom quark and antiquark are close to each other, asymptotic freedom comes into play, and the binding energy can be derived the same way we derive it for the hydrogen atom. To an approximation, there is a parallelism between the two physics [1]. For the singlet state the energy is

$$E=\frac{4}{9}\alpha_s^2 m_Q \tag{1}$$

and the radius is

$$a=\frac{3}{2\alpha_s m_Q}. \tag{2}$$

The wavelength of the gluon that dissociates this state fits the radius of the singlet state of Υ to a good approximation. The S matrix in this case is

$$S_{fi}=-i\int_{-\infty}^{\infty}dt\langle\text{octet}|grE^a\cos\theta|\text{singlet}\rangle, \tag{3}$$

where E^a is the color electric field. Taking the singlet state to be a $1s$ wave function and the octet to be a plane wave, and using the dipole moment matrix, the calculation of the S matrix in terms of the relative momentum of the $|b\overline{b}\rangle^{(8)}$ pair can be tedious but straightforward. The ionization of the hydrogen atom by electromagnetic radiation leads to a similar S matrix [1,2],

$$S_{fi}=\frac{32\pi gE^a(\omega)ka^5\cos\theta}{\sqrt{6}\sqrt{\pi a^3 V}(1+k^2a^2)^3}. \tag{4}$$

Here ω is the difference between the octet state energy and the singlet state energy, k is the relative momentum of the bottom quark in the octet state, a is the radius of the singlet state, and V is the quantization volume. Assuming color neutrality of the medium

$$\langle E_i^a(\omega)E_j^b(\omega)\rangle=\frac{1}{24}\delta_{ij}\delta_{ab}\langle|E(\omega)|^2\rangle. \tag{5}$$

Using Fermi's Golden Rule, the transition rate is

$$R_{fi}=2\pi\rho(k)|\langle\text{octet}|grE\cos\theta|\text{singlet}\rangle|^2, \tag{6}$$

where

$$\rho(k)=\frac{m_Q Vk\sin\theta d\theta d\phi}{8\pi^3}. \tag{7}$$

Integrating over k, the transition probability becomes

$$P_{fi}=\frac{2}{3}\pi\alpha_s a^2\langle|E(\omega)|^2\rangle. \tag{8}$$

There is a threshold energy of about 850 MeV for the dissociation of the upsilon meson into two highly energetic bottom quarks. Only gluons with energy exceeding $\omega_{\min}=850$ MeV can dissociate the upsilon. So the relevant energy density is not just the average energy density, but the energy of the gluons which have an energy higher than the threshold energy. In deconfined matter, such as quark-gluon plasma, we expect gluons in a medium of 200 MeV temperature to have an average momentum of 600 MeV [3]. So there are some higher-energy gluons which overcome the 850 MeV and dissociate the upsilon.

Dividing both sides of Eq. (8) by the total interaction time, τ_{fi}, I get

$$\Gamma_{\text{dis}}=\frac{2}{3}\pi a^2\alpha_s\frac{\langle|E(\omega)|^2\rangle}{\tau_{fi}}, \tag{9}$$

where $\langle|E(\omega)|^2\rangle/\tau_{fi}$ is the color-electric power density of the medium, which can be evaluated analytically for a variety of models. One such model is a dilute gas of color charges. Denoting the Casimir of the color charge by Q^2 in this model, it is found that

$$\frac{\langle|E(\omega)|^2\rangle}{\tau_{fi}}\approx\frac{\pi}{2}\alpha_s Q^2\tilde{\rho}(w), \tag{10}$$

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where $\tilde{\rho}(w)$ is the weighted average of the density of charges in the medium. Combining Eqs. (9) and (10), and introducing the number density of gluons, the dissociation rate of the upsilon meson becomes [1]

$$\Gamma_{\rm dis} \approx \frac{8}{9}\pi^3 \alpha_s^2 a^2 n. \tag{11}$$

This dissociation rate can be calculated in terms of the temperature of the quark-gluon plasma. But I will only include the gluons with energy exceeding the threshold energy of dissociation, $\omega_{\rm min}$. For a medium of gluons,

$$n = N/V = \frac{1}{2\pi^2}\int_{\omega_{\rm min}}^{\infty} d\omega \frac{\omega^2}{\exp(\omega/T)-1}, \tag{12}$$

which gives us

$$\tau_{\rm dis} \approx \frac{m_Q^2}{\pi \sum_{k=1}^{\infty}\left[\left(\frac{T}{k}\right)\omega_{\rm min}^2 + 2\left(\frac{T}{k}\right)^2 \omega_{\rm min} + 2\left(\frac{T}{k}\right)^3\right] e^{-k\omega_{\rm min}/T}}. \tag{13}$$

The minimum temperature required to achieve deconfinement is generally understood to be about 150–200 MeV. The Relativistic Heavy Ion Collider (RHIC) is the first collider designed to specifically create this plasma. It may reach a temperature of 500 MeV. CERN, on the other hand, hopes to reach a temperature of 1 GeV by colliding heavy nuclei in the Large Hadron Collider. Inserting a temperature of 500 MeV into Eq. (13), I get a dissociation time of 4 fm/c, which is comparable to the life span of the quark-gluon plasma—a typical life span of a quark-gluon plasma is about 2–5 fm/c [4]. If I use the CERN 1 GeV temperature, I get $\tau_{\rm dis} \approx 0.55$ fm/c (see Fig. 1).

In addition to the dissociation of upsilon by gluons, there is another kind of dissociation which is caused by the screening of the color charges of the quarks in the medium [5]. In the high-temperature deconfined phase, the bottom-antibottom free energy $V_{b\bar{b}}$, which is the Debye potential with inverse screening length m_{el}, is given by [6]

$$V_{b\bar{b}} = -\frac{4}{3}\frac{\alpha_s}{r} e^{-m_{el} r}. \tag{14}$$

Using a variational calculation with an exponential trial wave function, Ae^{-r/a_T}, I look for a critical value of m_{el} where the upsilon meson is no longer bound. I find this critical m_{el} to be

$$m_{el} = \frac{2}{3}\alpha_s m_Q. \tag{15}$$

Here I use [7] $\alpha_s(m_b) = 0.2325 \pm 0.0044$. Inserting m_{el} in the equation [6],

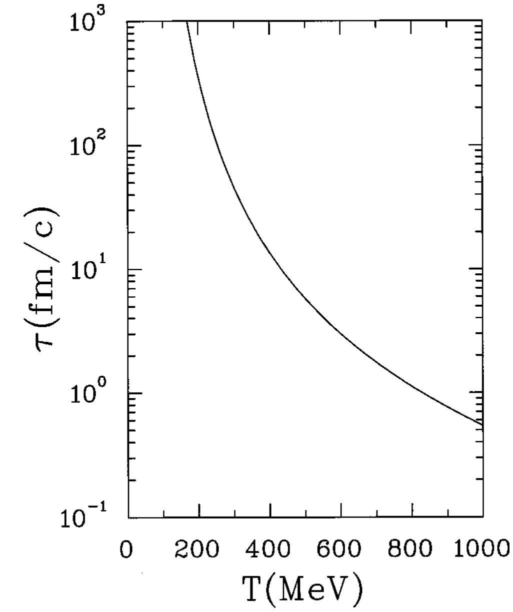

FIG. 1. Υ meson dissociation time in a quark-gluon plasma as a function of temperature.

$$m_{el}^2 = \frac{1}{3} g^2 \left(N + \frac{N_f}{2}\right) T^2, \tag{16}$$

where $N=3$ from the $SU(N)$ group and $N_f = 3$ is the number of light flavors, and using the temperature-dependent coupling constant as given by [6]

$$\frac{g^2}{4\pi} = \frac{6\pi}{27\ln(T/50\ \text{MeV})}, \tag{17}$$

I find that the ground state of upsilon is unbound at a temperature of $T = 250$ MeV. Above this temperature, the effect of screening does not allow the upsilon meson to exist in a 1s state.

According to the above results, the time of dissociation of the upsilon meson seems to be comparable to the life span of the quark-gluon plasma. If we calculate the dissociation time for J/ψ, using the same principle we used above, we will find it to be even less than the one for the Υ. This works better for the J/Ψ meson because its quark, c, is light compared to the b quark of the Υ, and also the binding energy of the J/Ψ is smaller compared to the binding energy of the Υ. Screening plays a major role when the temperature of the quark-gluon plasma exceeds 250 MeV. If there is a suppression of upsilon mesons, it is more likely due to the screening effect than it is to the high-energy gluon absoption or collision. I conclude that high-energy gluons and mostly the effect of screening are indispensable for the dissociation of the upsilon meson in the quark-gluon plasma.

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[1] B. Müller, nucl-th/9806023, v2.
[2] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968).
[3] D. Kharzeev and H. Satz, Phys. Lett. B **334**, 155 (1994).
[4] S. A. Bass *et al.*, Phys. Rev. C **60**, 021901 (1999).
[5] T. Matsui and H. Satz, Phys. Lett. B **178**, 416 (1986).
[6] J. I. Kapusta, *Finite-Temperature Field Theory* (Cambridge University Press, Cambridge, 1989).
[7] Matthias Jamin and Antonio Pich, Nucl. Phys. **B507**, 334 (1997).